



Oxford Cambridge and RSA

Thursday 25 May 2023 – Afternoon

A Level Further Mathematics A

Y540/01 Pure Core 1

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

1 In this question you must show detailed reasoning.

Determine the value of $\sum_{r=1}^{50} r^2(16-r)$. [3]

2 In this question you must show detailed reasoning.

The equation $z^4 + 4z^3 + 9z^2 + 10z + 6 = 0$ has roots α , β , γ and δ .

(a) Show that a quartic equation whose roots are $\alpha + 1$, $\beta + 1$, $\gamma + 1$ and $\delta + 1$ is $w^4 + 3w^2 + 2 = 0$. [3]

(b) Hence determine the exact roots of the equation $z^4 + 4z^3 + 9z^2 + 10z + 6 = 0$. [3]

3 (a) Show that $\frac{-3 + \sqrt{3}i}{2} = \sqrt{3}e^{\frac{5}{6}\pi i}$. [2]

(b) Hence determine the exact roots of the equation $z^5 = \frac{9(-3 + \sqrt{3}i)}{2}$, giving the roots in the form $re^{i\theta}$ where $r > 0$ and $0 \leq \theta < 2\pi$. [3]

4 The transformations T_A and T_B are represented by the matrices \mathbf{A} and \mathbf{B} respectively, where

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

(a) Describe geometrically the **single** transformation consisting of T_A followed by T_B . [2]

(b) By considering the transformation T_A , determine the matrix \mathbf{A}^{423} . [3]

The transformation T_C is represented by the matrix \mathbf{C} , where

$$\mathbf{C} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}.$$

The region R is defined by the set of points (x, y) satisfying the inequality $x^2 + y^2 \leq 36$.

The region R' is defined as the image of R under T_C .

(c) (i) Find the exact area of the region R' . [2]

(ii) Sketch the region R' , specifying all the points where the boundary of R' intersects the coordinate axes. [4]

5 (a) Find the general solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$. [2]

(b) Hence find the general solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = x(4 - 5x)$. [4]

6 **In this question you must show detailed reasoning.**

The power output, p watts, of a machine at time t hours after it is switched on can be modelled by the equation $p = 20 - 20 \tanh(1.44t)$ for $t \geq 0$.

Determine, according to the model, the **mean** power output of the machine over the first half hour after it is switched on. Give your answer correct to **2** decimal places. [4]

- 7 An engineer is modelling the motion of a particle P of mass 0.5 kg in a wind tunnel.

P is modelled as travelling in a straight line. The point O is a fixed point within the wind tunnel. The displacement of P from O at time t seconds is x metres, for $t \geq 0$.

You are given that $x \geq 0$ for all $t \geq 0$ and that P does not reach the end of the wind tunnel.

If $t \geq 0$, then P is subject to three forces which are modelled in the following way.

- The first force has a magnitude of $5(t+1)\cosh t \text{ N}$ and acts in the positive x -direction.
- The second force has a magnitude of $0.5x \text{ N}$ and acts towards O .
- The third force has a magnitude of $\left|\frac{dx}{dt}\right| \text{ N}$ and acts in the direction of motion of the particle.

- (a) The engineer applies the equation “ $F = ma$ ” to the model of the motion of P and derives the following differential equation.

$$5(t+1)\cosh t - 0.5x + \frac{dx}{dt} = 0.5\frac{d^2x}{dt^2}$$

- (i) Explain the sign of the $\frac{dx}{dt}$ term in the engineer’s differential equation. [1]

When $t = 0$ the displacement of P is 6 m , and it is travelling towards O with a speed of 5 ms^{-1} .

- (ii) Without attempting to solve the differential equation, find the acceleration of P when $t = 0$. [2]

Let the particular solution to the differential equation in part (a) be a function f such that $x = f(t)$ for $t \geq 0$.

The particular solution to the differential equation can be expressed as a Maclaurin series.

- (b) (i) Show that the Maclaurin series for $f(t)$ up to and including the term in t is $6 - 5t$. [1]

- (ii) Use your answer to part (a)(ii) to show that the term in t^2 in the Maclaurin series for $f(t)$ is $-3t^2$. [1]

- (iii) By differentiating the differential equation in part (a) with respect to t , show that the term in t^3 in the Maclaurin series for $f(t)$ is $0.5t^3$. [4]

You are given that the complete Maclaurin series for the function f is valid for all values of $t \geq 0$.

After 0.25 seconds P has travelled 1.43 m towards the origin.

- (c) (i) By using the Maclaurin series for $f(t)$ up to and including the term in t^3 , evaluate the suitability of the model for determining the displacement of P from O when $t = 0.25$. [1]

- (ii) Explain why it might not be sensible to use the Maclaurin series for $f(t)$ up to and including the term in t^3 to evaluate the suitability of the model for determining the displacement of P from O when $t = 10$. [1]

6

- 8 The points P , Q and R have coordinates $(0, 2, 3)$, $(2, 0, 1)$ and $(1, 3, 0)$ respectively.

The acute angle between the line segments PQ and PR is θ .

(a) Show that $\sin \theta = \frac{2}{11}\sqrt{22}$. [3]

The triangle PQR lies in the plane Π .

(b) Determine an equation for Π , giving your answer in the form $ax + by + cz = d$, where a , b , c and d are integers. [3]

The point S has coordinates $(5, 3, -1)$.

(c) By finding the shortest distance between S and the plane Π , show that the volume of the tetrahedron $PQRS$ is $\frac{14}{3}$.

[The volume of a tetrahedron is $\frac{1}{3} \times \text{area of base} \times \text{perpendicular height}$] [4]

The tetrahedron $PQRS$ is transformed to the tetrahedron $P'Q'R'S'$ by a rotation about the y -axis.

The x -coordinate of S' is $2\sqrt{2}$.

(d) By using the matrix for a rotation by angle θ about the y -axis, as given in the Formulae Booklet, determine in exact form the possible coordinates of R' . [5]

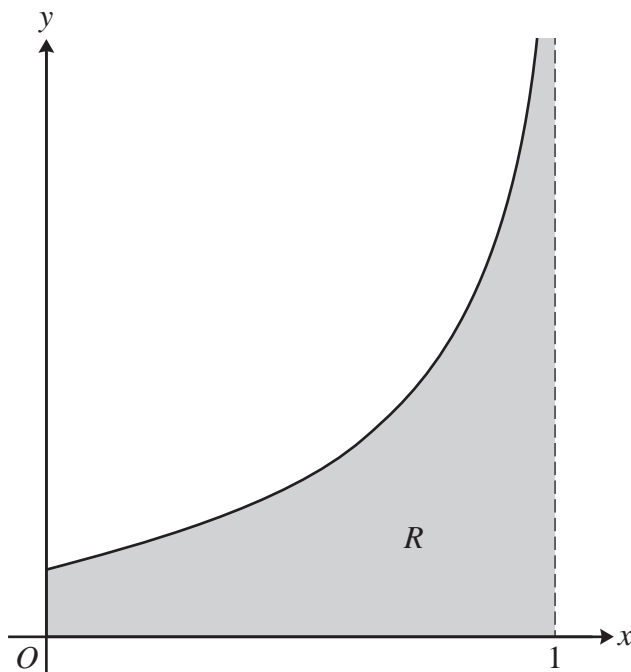
9 In this question you must show detailed reasoning.

- (a) Use de Moivre's theorem to determine constants A , B and C such that $\sin^4 \theta \equiv A \cos 4\theta + B \cos 2\theta + C$. [5]

The function f is defined by

$$f(x) = \sin(4 \sin^{-1}(x^{\frac{1}{5}})) - 8 \sin(2 \sin^{-1}(x^{\frac{1}{5}})) + 12 \sin^{-1}(x^{\frac{1}{5}}), \quad x \in \mathbb{R}, 0 \leq x < 1.$$

- (b) Show that $f'(x) = \frac{32}{5\sqrt{1-x^{\frac{2}{5}}}}$. [6]



The diagram shows the curve with equation $y = \frac{1}{\sqrt{1-x^{\frac{2}{5}}}}$ for $0 \leq x < 1$ and the asymptote $x = 1$. The region R is the unbounded region between the curve, the x -axis, the line $x = 0$ and the line $x = 1$.

You are given that the area of R is finite.

- (c) Determine the exact area of R . [3]

END OF QUESTION PAPER

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